Xu Guangqi and the Chinese Translation of Euclid's *Elements*

Some Problems of Terminology and Their Cultural Context

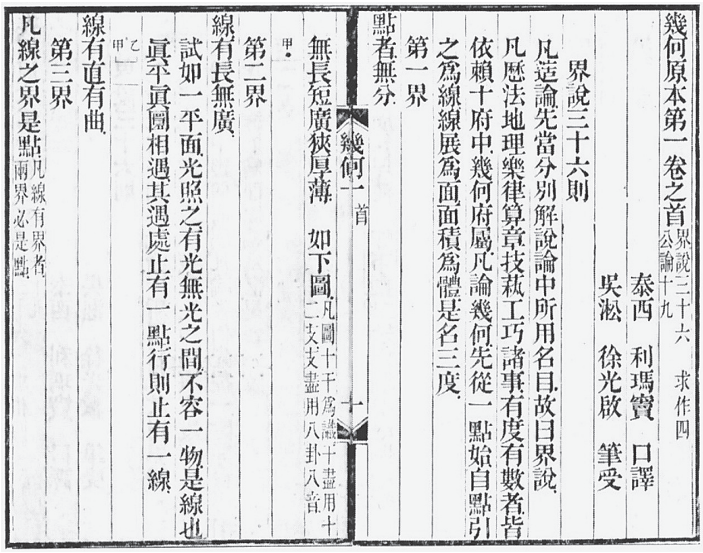
Masahiro OGAWA

1. Introduction: The Beginning of Western Studies in China

The first European classical work transmitted to Japan was the *Fables* of Aesop, whose Japanese translation, printed in Latin alphabet, was published in 1593 by the Christian college of Amakusa. Although it was fourteen years later, in 1607, that the Chinese people had their first translation of Western classical texts which had been brought into their country by a missionary of the same Roman Catholic Church, theirs was not a popular literary work like the *Fables* but the most important scientific text of ancient Greek mathematics, the *Elements* of Euclid (4th–3rd century B.C.). If one compares the two translations in respect of both content and form, one will be impressed not only by the great difference of intellectual level and difficulty between them but also by the fact that the latter was far better written in Chinese letters and in a noble style of classical Chinese. The first page of this version of the *Elements* shows its title and translators: 幾何原本 利瑪竇口訳 / 徐光啓筆受 (*Jihe yuanben*, dictated by Li Madou [Matteo Ricci] and transcribed by Xu Guangqi).[1]

Of the two translators Matteo Ricci (1552–1610) was an Italian Jesuit. He entered China under the Ming dynasty after the death of Francisco de Xavier (1506–1552) who, having left Japan, failed at landing on the continent of China. Ricci succeeded in his admission to the Chinese society and subsequently in his missionary work there, for he carried out an accommodating policy by learning well the Chinese language and classical literature and even by wearing Chinese clothes in the style of a Confucian scholar. Above all Ricci won a wide respect of the Chinese intellectual class by means of writing in Chinese on the Christian religion (天主実義, *Tianzhu shiyi* [*The True Meaning of the Lord of Heaven*]) and publishing various scientific works including a

1 The text of the *Jihe yuanben* is quoted in this article from *Xu Guangqi quanji* (徐光啓全集), vol. 4, Shanghai, 2010. The first two pages inserted in the next page were taken from a reprinted edition of the *Jihe yuanben*, 1865.


幾何原本第一卷之首 界說三十六 公論十九 求作四
泰西 利瑪竇 口譯
吳淞 徐光啟 筆受
界說三十六則
凡造論先當分別解說論中所用名目故曰界說
凡歷法地理樂律算章技藝工巧諸事有度有數者皆
依賴十府中幾何府屬凡論幾何先從一點始自點引
之為線線展為面面積為體是名三度
第一界
點者無分
幾何一 首 十
無長短廣狹厚薄 如下圖 凡圖十干為識十盡用十二支支盡用八卦八音
甲
第二界
線有長無廣
試如一平面光照之有光無光之間不容一物是線也
真平真圓相遇其遇處止有一點行則止有一線
乙 甲
線有直有曲
第三界
凡線之界是點 凡線有界者兩界必是點

world map named 坤輿万国全図 (*Kunyu Wanguo Quantu*) and an astronomical treatise entitled 乾坤体義 (*Qiankun tiyi*).[2] Of these numerous contributions by Ricci the translation of Euclid's *Elements* was most likely to exert a significant influence on later culture of China, because it was an essential work that had constituted the foundation of Western science.

But, however well-acquainted with the Chinese cultural tradition and language, Matteo Ricci could not have made a Chinese translation of the *Elements* without a competent native partner.[3] Fortunately for the Jesuit order as well as for the Chinese society, he met a most open-minded and able cultured person in late Ming China, whose name was Xu Guangqi. Only with the ardent support and cooperation of Xu he could make the essence of classical Greek mathematics known to Eastern Asia. If Matteo Ricci can be called "the first Man of the World",[4] having studied Chinese classics earlier than any Westerner, Xu Guangqi must be surely said to have been, to contemporary Western people, "the first Chinese to have a face",[5] for he was the

2 Ricci also translated Chinese classical literature such as *The Four Books of Confucianism* (四書) into Latin: cf. S. Hirakawa (平川祐弘), *A Biograpy of Matteo Ricci* (マッテオ・リッチ伝), vol. 1, Tokyo, 1969, pp. 123–4, 126, 132, 141, 260.

3 Before meeting Xu Guangqi Ricci translated with Qu Taisu (瞿太素) the first book of the *Elements*, but this first version of the *Jihe yuanben* is not extant: cf. P. M. Engelfriet, *Euclid in China. The Genesis of the First Chinese Translation of Euclid's Elements Books I–VI (Jihe yuanben; Beijing, 1607) and its Reception up to 1723*, Leiden, 1998, pp. 59ff. Ricci also published in cooperation with Li Zhizao (李之藻, 1565–1630) a mathematical book (同文算指 [*Tongwen suanzhi*]) and an astronomical work (渾蓋通憲図説 [*Hungai tongxian tushuo*]). The collection of works entitled *Tianxue chuhan* (天学初函), edited later by Li Zhizao, contains many Christian and scientific books translated by Ricci together with his Chinese partners (including the *Tianzhu shiyi* and the *Jihe yuanben*).

4 "Saisho no sekai-jin" (最初の世界人): a comment by S. Hirakawa in the Japanese translation of Ricci's *De Christiana Expeditione apud Sinas* (中国キリスト教布教史), vol. 1, Tokyo, 1982, pp. 629–30.

5 C. Jami, P. Engelfriet & G. Blue (eds.), *Statecraft and Intellectual Renewal in Late Ming China. The Cross-Cultural Synthesis of Xu Quangqi (1562–1633)*, Leiden, 2001, Introduction, p. 1.

earliest man in China to turn his front directly to European culture. One may then possibly ask why such a happy encounter did happen in China but never in Japan where the Jesuits came almost at the same period. To this interesting hypothetical question, we may give a reasonable answer not by assuming that there was not a "Man of the World" like Ricci among the Jesuits who visited Japan at an early period, but rather by admitting frankly that there was actually a definite gap between Japanese and Chinese traditional intellectual cultures at that time. In fact, as Xavier's later course of interest suggests, any tradition of original spiritual culture could not be found then in Japan, and even if some foreigner wished to learn any Japanese culture, it proved generally to be traced back to Chinese origins.

Ricci's co-translator Xu Guangqi (1562–1633) was born in Shanghai and, after having passed at the age of forty-two the final imperial examination (進士, jinshi) destined to higher officialdom, became a member of the Hanlin Academy (翰林院), an academic institution at which an elite group of scholars studied and worked for the emperor's politics and administration. In his subsequent career until death as an imperial officer and minister Xu devoted considerable efforts to saving and reinforcing the Ming dynasty faced to national crises, working very hard in particular for the reforms of calendar, agriculture and military affairs.[6] It was in 1600 that he met Ricci for the first time in Nanjing (南京), and two years later he became a Christian (his baptismal name was Paulo 保禄). Ricci and Xu seem to have been engaged in the translation of Euclid for one year and a half from 1606. It is said that, while serving at the Hanlin Academy in Beijing, Xu visited Ricci's residence every day to have a long discussion on the text of the *Elements* and continued to render everything he had learned into a "lucid, imposing and elegant style", until they finished translating "the first six books which are the most necessary".[7]

The text on which they based their translation was a Latin version (*Euclidis Elementorum Libri XV*, the first edition, 1574)[8] edited by Christopher Clavius, who was a prominent mathematician at that time and had taught Matteo Ricci at the Jesuit Collegio Romano in Rome. As Xu Guangqi could not understand Western languages, he must have listened attentively to Ricci's exposition in rough Chinese and asked him about every obscure point each time before putting some amount of Latin passages into a smooth, traditional style of Chinese. To perform such a highly intellectual and technical job as the translation of the *Elements*, he had to coin so many new terms and to select carefully the accurate expressions of mathematical concepts and contexts from the wide vocabulary of Chinese. Although the *Jihe yuanben* simply says at the beginning that it was "transcribed" (筆受) by Xu Guangqi, one will easily suppose that this kind of work could not be accomplished by an ordinary court scholar

6 On the social and cultural conditions of late Ming China and Xu Guangqi's life and achievements: cf. S. Okamoto (岡本さえ), Xu Guangqi and the "Barbarians" (徐光啓と「夷狄」), in: S. Hirokawa (ed.), *People who lived in Different Cultures* (異文化を生きた人々), Tokyo, 1993, pp. 47–71; M.-K. Siu, Success and Failure of Xu Guangqi: Response to the First Dissemination of European Science in Ming China, *Studies of History of Medicine & Science* 14 (1995/96), pp. 137–79.

7 Quotations of Matteo Ricci's words in his *De Christiana Expeditione apud Sinas* (*op. cit.*, n. 4), vol. 2, pp. 72, 73.

8 A later version is included in: Ch.Clavius, *Opera Mathematica*, 5 vols, Moguntia, 1612.

specialized alone in making literary compositions, but only by an exceptional figure having also an outstanding ability of logical thinking as well as a genuine interest in different culture and science.

When they worked for one year and a half and revised three times their manuscript of the first half, Xu Guangqi suggested to Matteo Ricci that they should continue their job to the end of the *Elements*. But Ricci wanted to stop there, in order to make Xu rest for a while and to see what effects their partial translation would exercise on Chinese intellectuals.[9] Thus in 1607 the first six books of the *Jihe yuanben* were published with Ricci's preface (訳幾何原本引) and Xu's prelude (幾何原本雑議). Unfortunately Ricci passed away after three years, so that the complete version of Euclid did not appear until as late as two and a half centuries afterwards. It was in 1859 that a British missionary Alexander Wylie and a Chinese mathematician Li Shanlan (李善蘭) published their translation of the remaining nine books of the *Elements*.

I have drawn above a rough sketch of how the *Jihe yuanben* became the first translation of Western books in China. In the following chapters, I shall discuss first what problems Ricci and Xu found and how they attempted to resolve them when translating the mathematical terms, especially those of the most fundamental concepts in the *Elements*. Next I shall consider how Xu Guangqi interpreted and evaluated the whole text of Euclid from the viewpoint of a Chinese official and intellectual, and then what effects his interpretation and evaluation brought about on later culture of China. It will be of great interest to notice some feature of Chinese culture clearly marked in a series of reactions which were caused by the Western classical text as a sort of catalyst. Lastly this interesting observation of the Chinese reception of European scientific studies will make us turn, for contrast, to a brief historical reflection on Japanese modern science and culture.

2. The Text of the *Jihe yuanben* and the Translation of Basic Terms

As is well-known, the most dominant characteristic of Euclid's mathematical work is that it has a unique system built up by a strict deductive method. At the beginning the principles (definitions, postulates, common notions or axioms) are stated as undoubted premises which must be accepted without any proof. After that the propositions are given successively and demonstrated logically by applying various principles. The propositions thus proved can serve hereafter, together with the principles, as solid foundations for the subsequent proofs of other propositions, which will become more and more complicated. Another important feature of the *Elements* can be noted in the whole structure: the theory of geometry (plane geometry in books 1–6 and solid geometry in books 11–13) is quite carefully set apart from that of number (integral number in books 7–9 and irrational number in book 10). Moreover, calculations of numbers are not employed in the proofs of the propositions of plane and solid

9 Ricci, *op. cit.* (n. 7), p. 73.

geometries. This means that, by drawing a clear distinction between the two parts concerning continuous quantity (magnitude) and discrete quantity (number), Euclid planned for his construction of mathematical system to make the geometrical studies as independent as possible from arithmetical operations. The total six books of the *Jihe yuanben*, then, deal only with the theory of plane geometry.

The Greek text of the *Elements* commonly used today has been divided into thirteen books and based on a modern edition (1883) by J. L. Heiberg,[10] which depended directly on the Vatican manuscript discovered by F. Peyrard in the early nineteenth century. On the other hand, Clavius' edition in Latin, which served as the basic text of the *Jihe yuanben*, consisted of fifteen books.[11] Clavius revised the Latin version created from the Greek text made by Theon of Alexandria (4th century A.D.) and from the medieval Arabic translation, adding his abundant commentaries to the text and including the last two books falsely attributed to Euclid. It is therefore necessary to have both the current Greek text and Clavius' Latin edition, when one is to think about the problems of Chinese translation of the *Elements*.

The main differences between the current Greek text and the *Jihe yuanben* based on Clavius' edition can be enumerated as follows: in the first book of the latter the subdivision and addition of definitions resulting in the increase of their number from twenty-three to thirty-six, the transference of two postulates to common notions, the addition of one new postulate and the rearrangement of five common notions into nineteen, and in the fifth and sixth books the addition of some definitions.[12] But what merits most our attention at present is some inevitable difficulties the translators had to face in finding out adequate Chinese words for the basic concepts in Euclid. As the title *Jihe yuanben* itself is to be discussed later, I shall start with the principles described in the first book. In the following citations, (1) Chinese translation in the *Jihe yuanben*, (2) Latin translation by Clavius, (3) Romanized Greek original words will be listed.

"Definition"
(1) 界説 (Jieshuo)
(2) Definitio
(3) Horos

The first page of the original text of the *Elements* has not any preface to show the author's purpose, intention or preliminary remarks, but begins quite abruptly by enumerating the definitions of various geometrical figures. Besides, any explanation of the idea of definition itself is not given there, so that the Chinese translation, after putting a neologism 界説 at the head, adds a short note: "When making up a theory, it is necessary first to sort out and explain the names used in the theory. So we say 界説". But why should one call by 界説 the explanation of the names of mathematical figures? The reason does not seem yet to have become clear enough to stop average Chinese readers from wondering at this new term. Moreover, in every part of the *Jihe yuanben*,

10 E. S. Stamatis (ed.), *Euclidis Elementa* (post J. L. Heiberg), Leipzig: Teubner, 1969–.
11 On Clavius' edition: cf. Engelfriet, *op. cit.* (n. 3), pp. 105–31.
12 Cf. Tae-ok An (安大玉), *Minmatsu Seiyō Kagaku Tōdenshi* (明末西洋科学東伝史 [*History of the Transmission of Western Science to Late Ming China*]), Tokyo, 2007, pp. 65–76.

界説 is abridged to 界 to indicate each definition (for example 第一界 stands for *Definition 1*), while some geometrical elements such as point, line and circumference are also referred to as 界 in certain passages. This way of using the word 界 looked to be so confusing that the problem has been pointed out by some specialists.[13] It is, however, possible to guess that the translators, after meeting such a difficulty as of picking up a correct word for the quite new concept, dared to choose a new, unfamiliar term, no matter how ambiguous meaning it might convey according to contexts.

In its primary sense "definition" means demarcating a thing or an idea and separating it from another. For this reason Euclid used for it a Greek word "horos" which signifies "boundary, limit, border". In the Latin version too, this Greek word was translated as "definitio" which derived from "finis", a Latin word denoting "boundary, limit, border" and whose basic idea was then "marking the edges". It is therefore obvious that Ricci and Xu adopted an almost literal translation 界説 (界 [limit, boundary] and 説 [expounding]) to respect the sense of the original word. But 界説 is not only a precise Chinese expression for "definition", clearly implying an etymological significance of Greek and Latin. It is also closely connected with the content of each definition in the first book. In *Definition 13* the word 界 itself is defined as follows:

Definition 13
(1) 界者、一物之始終。(A boundary is the end or beginning of a thing.)
(2) Terminus est, quod alicuius extremum est.
(3) Horos estin, ho tinos esti peras.

In this sentence 界 is presented formally as a Chinese word for "terminus" (boundary), a Latin word which carries the literal meaning of the Greek "horos" (boundary). In the predicate the words 始終 (end or beginning) correspond to the Latin "extremum" (extremity) and the Greek "peras" (extremity), probably because an alternative word 端, the Chinese equivalent to "extremity", is a word susceptible of standing also for "cause" or "origin".

Definition 3
(1) 線之界是点。(The extremities of a line are points.)
(2) Lineae autem termini, sunt puncta.
(3) Grammes de perata semeia.

Here, too, 界 represents the idea of boundary (terminus), which accords perfectly with *Definition 13*. In the Latin version "terminus" stands for the Greek "peras", not "horos", since in *Definition 13* "terminus" is defined as "extremum" which is a Latin translation of "peras", the Greek word defining "horos".

Definition 6
(1) 面之界是線。(The extremities of a surface are lines)
(2) Superficiei autem extrema, sunt lineae.

13 Engelfriet, *op. cit.* (n. 3), pp. 147, 206; An, *op. cit.* (n. 12), pp. 82–3.

(3) Epiphaneias de perata grammai.

界 means here "extremum", which may look to be inconsistent with *Definitions 13* and *3*. However, as "terminus" is defined as "extremum" in *Definition 13*, "extremum" can be translated as 界 (=terminus) without any problem. This is the same way as Clavius translated "horos" (defined by peras) as "terminus" in *Definition 13*, while he put "peras" into the same word "terminus" in *Definition 3*.

Definition 14

(1) 或在一界或在多界之間為形。
(Either within one boundary or more boundaries is a figure.)
(2) Figura est, quae sub aliquo, vel aliquibus terminis comprehenditur.
(3) Skhema esti to ypo tinos e tinon horon periekhomenon.

Just as in *Definition 13*, 界 is used here as an expression of exactly the same sense as "terminus" and "horos".

In other definitions of the *Jihe yuanben* 界 appears as a word indicating a line which shapes circle, diameter, semicircle, rectilineal, trilateral, quadrilateral and multilateral figures (*Definition 15, 17, 18, 19, 20, 21, 22*). Although in these cases 界 corresponds to different Latin words, it always means "boundary" or "limit". Particularly in the first parts of definitions, we have seen that what is meant by 界 is actually the Latin "terminus" and its synonym "extremum", which in turn were the translations of the Greek "horos" or its synonym "peras". In this way, the idea of "boundary" is constantly implied by 界 in the Chinese definitions and it is well worth remarking that it is exactly those "boundaries" which are working for "defining" a whole or a part of each geometrical figure in the original text.

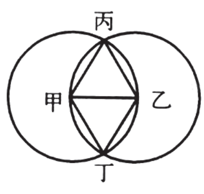
It is to be noted, on the other hand, that the Latin word "finis" from which "definitio" (definition) derived is also synonymous with "terminus". If various figures take their proper form by being marked with 界 (boundary), the "exposition of boundaries", 界説, should be regarded as fundamental to the definitions of geometrical figures. Certainly Euclid wrote nothing more than a word "horoi" (boundaries, the plural of "horos") at the very start of formulating his long theories. This opening word, without any explanation, is too simple, but it is none the less intended that its meaning should be understood by watching, in each definition, how a figure is going to be shaped with "horos" or "horoi". The Latin version also reflected faithfully this original intention, and having read closely a typical Euclidean "geometrical" thinking in the Latin text, the translators in Chinese chose a new coinage of 界説 which bears a geometrically rich implication. If someone says that 界説 is not an appropriate translation, he must be, then, regarded not as reproaching Ricci and Xu but rather as offering a critical comment on the scientific thought of Euclid himself.

The new term 界説 was therefore created through a careful and correct reading of the original text of the *Elements*. But curiously enough, in later times, 界説 was wholly neglected in East Asian countries and, instead, a Japanese word 定義 (teigi), which was coined in the Meiji era by paraphrasing the English "definition", has swept over China and Korea as well as Japan. For this subsequent issue, on which many factors other than academic conditions might have had a considerable effect, a part of responsibility, in my opinion, should be lying with Xu Guangqi. I shall return to this problem in the next chapters.

"Common notion" or "Axiom"
(1) 公論 (Gonglun)
(2) Communis notio, Axioma
(3) Koine ennoia

In the first book, after "definitions", there come "postulates" and "common notions". "Postulate" ("aitema" in Greek and "postulatum" or "petitio" in Latin) is translated as 求作 (qiuzuo) and consists of four principles for the constructions of straight line, circle and so on. "Common notion" represents, on the other hand, a fundamental proposition which does not need to be proved or a statement generally accepted as true: for example "Things which are equal to the same thing are equal one to another", or "If equals are added to equals, the wholes are equal". The alternative Latin "axioma", a loanword from Greek, signified in general "a matter which is thought right and suitable" and especially in Greek philosophy and science "a self-evident principle", whose meaning the modern word 公理 (gongli, "commonly accepted truth") conveys well. But the Chinese word 公論, which means generally "fair discussion" or "public opinion", does not fit properly the sense of the original word. So one will wonder legitimately what idea the translators exactly had of this diction.[14] Actually, as they had to write at the head a short note warning that "gonglun should not be doubted" (公論者不可疑), they appear to have felt a little uneasy about this somewhat forced translation. Why did Ricci and Xu choose as a word for "common notion" or "axiom" 公論, which had a potential for causing problems, rather than 公理, a more adequate word that was adopted by later mathematical studies in China, Korea and Japan?[15] Keeping this question in mind, I shall now see how the proof of proposition is described in the first book of the *Jihe yuanben*.

第一題 (*Proposition 1*)
于有界直線上、求立平辺三角形。
(On a limited straight line it is asked to make an equilateral triangle.)



法曰、甲乙直線上、求立平辺三角形。先以甲為心、乙為界、作丙乙丁円。次以乙為心、甲為界、作丙甲丁円。両円相交于丙、于丁。末自甲至丙、丙至乙各作直線、即甲乙丙為平辺三角形。

14 Cf. Engelfriet, *op. cit.* (n. 3), pp. 148–9, 206.
15 Cf. An, *op. cit.* (n. 12), p. 112.

(**"Fa" says**: It is asked to make an equilateral triangle on the straight line 甲乙. First make a circle 丙乙丁 with 甲 as center and 乙 as border. Next make a circle 丙甲丁 with 乙 as center and 甲 as border. The two circles cross each other at 丙 and 丁. Lastly make straight lines from 甲 to 丙 and from 丙 to 乙. Then 甲乙丙 makes an equilateral triangle.)

論曰、以甲為心、至円之界、其甲乙線与甲丙、甲丁線等。以乙為心、則乙甲線与乙丙、乙丁線亦等。何者。凡為円、自心至界、各線俱等故。既乙丙等于乙甲、而甲丙亦等于甲乙、即甲丙亦等于乙丙。三辺等、如所求。

(**"Lun" says**: As from the center 甲 to the border of circle, the line 甲乙 equals the line 甲丙 and the line 甲丁. As from the center 乙, the line 乙甲 equals the line 乙丙 and the line 乙丁. Why ? For all lines from the center to the border of circle equal each other. Already 乙丙 equals 乙甲, and 甲丙 equals 甲乙. So 甲丙 also equals 乙丙. The three sides are equal. It is what was asked.)

In the manuscripts of Euclid's *Elements* propositions were simply put side by side, headed only with reference number and without any title and classification. On the other hand, in the Western translations, they have been generally sorted out by their content into "theorems" and "problems", so that in Clavius' edition each "propositio" is marked either with "theorema" or "problema". The *Jihe yuanben* differs from Clavius' edition in having no explicit indication of this distinction but, instead, shows the difference by the two forms of subdivided structure. It follows for theorems the process of "題 (ti)—解 (jie)—論 (lun)" and for problems that of "題 (ti)—法 (fa)—論 (lun)", in which 題 means a proposition, 解 an explanation of a theorem, 法 a method to pose a problem, and 論 a demonstration.[16]

In the part of 論 quoted above, there are also some short references carefully added to the main text, such as "界説 15" after "For all lines from the center to the border of circle equal each other" and "公論 1" after "So 甲丙 also equals 乙丙", thus helping to understand the logical sequences. It is of course within the part of 論 that a proposition is to be established definitely and beyond dispute through a very rigorous deduction from evident principles so as to offer itself subsequently as a firm basis to higher demonstrations. But, just as in the case of 公論, a question has to be raised here why what must constitute the central section of each theoretical statement of the *Elements* should be translated as 論, a common word which carries a wide range of connotations, signifying "discussion", "dispute", "criticism", "argument", "comment", "judgement", "opinion" and "discourse".[17] In addition to that, 公論 (common notion,

16 Clavius' text does not give an indication of the division like "*jie* or *fa—lun*". As An suggests (*op. cit.* p. 78), this subdivision seems to have been invented by the translators well aware of the traditional writing style of Chinese mathematics. But it has to be noted that in old Chinese mathematics there was no demonstration (*lun*) such as in Euclid: see the next chapter.

17 Cf. J.-C. Martzloff, Eléments de réflexion sur les réactions chinoises à la géométrie euclidienne à la fin du XVII^{e} siècle—Le *Jihe lunyue* de Du Zhigeng vu principalement à partir de la préface de l'auteur et deux notices bibliographiques rédigées par des lettrés illustres, *Historia Mathematica* 20 (1993), pp. 163–4; P. Engelfriet, The Chinese Euclid and its European Context, in: C. Jami & H. Delahaye (eds.), *L'Europe en Chine. Interactions scientifiques, religieuses et culturelles aux XVII^{e} et XVIII^{e} siècles*, Paris, 1993, p. 121; Engelfriet, *op. cit.* (n. 3), pp. 149–50, 206.

axiom) is occasionally abbreviated to the same word 論. Due to these problems concerning the most important term, it must have become more difficult for many Chinese readers to have a proper understanding of the *Jihe yuanben*.

For a discussion on such a philological sort of question, it does not seem to be so fruitful only to point out the translators' drawbacks, but it is also necessary to read from the common reader's (accordingly the translators') point of view. So if one reads the Chinese text without preconception, then, 論 of 公論 and 論 of the propositions, just like 界 (說) and 界, can be understood as the same word of the same sense. From this simple reading it would be logical to interpret both *lun*s as implying "proof" or "demonstration". In this sense the identity of the two *lun*s might be considered not as the translators' careless confusion of different ideas, but rather as resulting from their intention to integrate them. In its proper context 公論 means surely a proposition which need no proof. But, since it stands for a matter commonly accepted as true and evident, 公論 is indispensable to the deductive demonstration and, conversely, it cannot exist or is useless (at least for Euclid's mathematics) if separated from the purpose of proving. This inseparable relation between axioms (公論) and proofs (論), like an unbroken chain, forms an essential feature of Euclid's logic of deduction, with which a proposition, when demonstrated by axioms, is never closed then but, given a permanent and universal value, will be combined with them to supply another proposition to be demonstrated in a more complicated manner. Ricci has told of this characteristic of the logic peculiar to the *Elements* in his preface:

> Before the propositions (題) and the demonstrations (論), the definitions (界說) have been laid down. After that, the axioms (公論) have been formulated on which the propositions and the demonstrations rest. Next, the propositions follow. They give a first explanation (解), a manner of construction (作法) and a proof (推論). What comes later is based on the results that have been obtained before. The more than five hundred propositions, divided over thirteen books, unroll themselves in a straight line, connected with each other, from beginning to end. Nowhere can their order be reversed. They go on piled and combined, without any pause until the last one. The principles at the beginning are very reasonable, extremely simple and clear. But they are gradually accumulated and reveal at last the hidden meaning.[18]

At the beginning of the same preface Ricci has also referred in a more general manner to the method of Western thinking which crystallized into Euclidean mathematics:

> 彼士立論宗旨、惟尚理之所抝、弗取人之所意。[19]
> (The scholars in the West, as fundamental principles to make a theory, only respect what has been proved by reason, and they never adopt what men only suppose.)

18 *Xu Guangqi quanji* (徐光啓全集), vol. 4, p. 9.
19 *Id.*, p. 6.

What has been told here as "a theory" (論) points to "demonstrations" (論) in the former passage quoted above, while "fundamental principles" (宗旨) correspond to "axioms" (公論) and the proved "propositions" (題). So, in these sentences, it is strongly suggested that in the translated text 公論 is a term connoting "what has been proved by reason" and never "what men only suppose".

In this way 論 is often used as a word implying the rigid demonstration by reason. But it is nevertheless true that this meaning has diverged from the normal usage of the word. To make the problem more serious, Ricci has mentioned Clavius' 論 as his teacher's achievements in the last part of his preface:

> Master Ding [Clavius], who was thoroughly familiar with this work, already wrote a commentary on it. Moreover, he produced two books with further considerations and additions. Together with the original work, there are totally fifteen books. He has also created new theories (新論) based on the same principles in each book. Thus this work became more detailed and complete.[20]

In Clavius' edition propositions are often provided with his own proofs besides those of Euclid, whereas the *Jihe yuanben* offers only the original proofs. Of course Clavius' additions must be treated as his commentaries distinct from the text itself, so that Ricci and Xu generally obeyed this rule in their translation.[21] But it would be another matter to inform the readers of Clavius' additions to the text as 新論. It might produce some confusion, for, even if Clavius' proofs might have seemed to be more adequate in some cases, the readers, who were not given at all any further information of what the "new theories" meant, could have an impression that 論 was a sort of argument drawn from "what men only suppose", not an exact demonstration "by reason". Xu Guangqi has said in his prelude to the translation: "There are four things unnecessary in this work: it is unnecessary to doubt, unnecessary to guess, unnecessary to test and unnecessary to correct. There are four things impossible: it is impossible to escape, impossible to refute, impossible to reduce and impossible to change the context".[22] If this is an effective explanation of how unparalleled Euclid's demonstration is in lucidity and accuracy, the reference to Clavius' "new theories" would make nothing but blur a clear image of this strict mathematical system.

In the translators' view 論 had to mean the demonstration by a deductive method. Furthermore, they wanted to apply this interpretation to the axiom by naming it as 公論. Yet it cannot be still denied that they performed a sort of tour de force by straining to some extent the meaning of the Chinese word, and we have seen that they were probably aware of their "performance" in the case of 公論. The question is, therefore, to be formulated again: why did Ricci and Xu persist in their interpretation in spite of its linguistic difficulty? But, at the moment, before seeking a solution to this question

20 *Id.*, p. 10.

21 Ricci states in his preface to the *Jihe yuanben* (*id.*, p. 11): "Euclid's text has been translated without omission. Master Ding's commentary has been limited to the first demonstrations" (但欧几里得本文、已不遺旨、若丁先生之文、惟訳註首論耳).

22 此書有四不必。不必疑、不必揣、不必試、不必改。有四不可得。欲脱之不可得、欲駁之不可得、欲減之不可得、欲前後更置之不可得。(*id.*, p. 13)

we shall have to examine another basic term.

Among the new words coined in the *Jihe yuanben*, there are quite many which are still in use as technical terms of geometry broadly in China, Korea and Japan, such as 点 (point), 線 (line), 直線 (straight line), 曲線 (curve), 対角線 (diagonal), 平行線 (parallel lines), 直角 (right angle), 鋭角 (acute angle), 鈍角 (obtuse angle), 面 (surface), 平面 (plane surface), 三角形 (triangle), 面積 (area), 体積 (volume).[23] These are mainly the words concerning individual geometrical figures and their parts. In a sharp contrast with them, the terms regarding the fundamental structure of Euclid's deductive demonstration, such as 界説, 求作, 公論, 題 and 論, have not been employed long since and were replaced by the Japanese words 定義 (teigi), 公準 (koujun), 公理 (kouri), 命題 (meidai) and 証明 (shoumei) respectively.[24] These conflicting facts seem to suggest that, whereas the *Jihe yuanben* actually played a leading part in the diffusion of Western geometrical studies in Eastern Asia, the understanding and recognition of its mathematical theory founded on a deductive method did not grow among the Chinese against the translators' expectation. On the other hand, the greatest linguistic merit of this translated work everyone in the three countries has to acknowledge is, undoubtedly, the creation of the word 幾何 as a general name disignating geometry.

There are a few hypotheses about the origins of 幾何 (jihe) of the *Jihe yuanben*.[25] The most frequently mentioned until some time ago is the one which supposes "jihe" to be a transliteration of the first sounds "geo" of geometry. This conjecture, although interesting and still supported by some specialists, is not well founded, especially because neither the title of Euclid's work (*Stoicheia*) nor that of its Latin version (*Elementa*) includes any word like "geometria". So we have to look for a clue elsewhere.

At the beginning of the *Jihe yuanben* an important note is written: "In the studies of calendar, geography, music, arithmetic, arts and crafts, the matters concerning magnitude (度) and number (数) belong to the category of *jihe* (幾何)". In this notice it is clearly shown that "jihe" (幾何) means not only the learning of figures (geometry) but in fact the mathematical studies in general which deal with all the quantities consisting of both "magnitude" and "number" and, in China, used to be called 度数之学 (study of magnitudes and numbers). As already mentioned, the *Elements* is actually composed of both studies of continuous quantity (magnitude) and discrete quantity (number), so that this definition of "jihe" entirely agrees with the whole content of the original text. But, incidentally, the *Jihe yuanben* was not completed until much later and remained for a long time a translation of only the first six books about plane geometry. These accidental conditions would have caused a general misunderstanding, with the result that "jihe" has become the term standing solely for geometry.

23 Cf. Li Yan & Du Shiran, *Chinese Mathematics. A Concise History*, Oxford, 1987, p. 194; Engelfriet, *op. cit.* (n. 3), pp. 282–6; An, *op. cit.* (n. 12), pp. 112–3.

24 Cf. An, *op. cit.* (n. 12), pp. 112–3. Only in China 公設 has been used instead of 公準.

25 On the title *Jihe yuanben*: cf. Martzloff, art. cit. (n. 17), p. 163; Siu, art. cit. (n. 6), pp. 160–1; Engelfriet, *op. cit.* (n. 3), pp. 138–42; An, *op. cit.* (n. 12), pp. 98–100; J. Watanabe, On the Etymology of 'Ji-he' from the Viewpoints of Chinese and Manchu Linguistics, *RIMS Kôkyûroku* 1444 (2005), pp. 34–42.

The question why *mathematics* could be called "jihe" seems to have been settled today. The answer, quite easy to understand, is that it was an everyday Chinese word meaning "how much, how many". The colloquial expression "jihe" could be used for magnitude as well as number. So, not only plain but correct, this must be said to have been a truly excellent name for mathematics. Moreover, this word was also found in Chinese classical texts as a kind of formula. For example, the *Jiuzhang suanshu* (九章算術 [*The Nine Chapters on the Mathematical Art*]), the most important ancient work of Chinese mathematics, begins with sentences such as: "Now there is a rice field 15 feet wide and 16 feet long. Question: what is the measure (*jihe*) of the rice field?—Answer: 240 tsubo". It is almost certain that Ricci and Xu had in mind this typical form of question and answer (問 X 幾何—答曰 Y) in traditional Chinese mathematics when they adopted "jihe" for their book title.[26]

If "jihe" (幾何) stands for mathematics, what sense is conveyed by "yuanben" (原本)? It may mean "foundation" or "book of origin". But in his preface Ricci has explained briefly: "The reason for calling the book "*yuanben*" is that it clarifies why mathematics gets what it gets (曰原本者、明幾何之所以然)"[27]. This explanation does not only show that the *Jihe yuanben* represents in a general and abstract manner a work expounding the basis or root of mathematics.[28] "Yuanben" should be regarded as a translation of the original title of *Stoicheia* or *Elementa*, which was interpreted as what "clarifies why X gets what it gets". Since "X" had been omitted in Euclid's title, what matters and impresses most with the Chinese title is that it expressly added and combined to "yuanben" a unique word symbolizing *Chinese* mathematics, that is, "jihe" (幾何). The *Jihe yuanben* is thus a very challenging title in itself, for it conveys and asserts a firm intention of adapting Western mathematics to Chinese context. In other words, this title can be understood as a clear manifestation of affording indubitable evidence to the traditional Chinese mathematics which developed from old times with a form of answering to the question "jihe?" (幾何).

3. Xu Guangqi's Interpretation and its Influence

Around the years of translating the *Jihe yuanben* Xu Guangqi published, besides a preface (刻幾何原本序) and a prelude (幾何原本雜議) to this work, the *Celiang fayi* (測量法義 [*Explanations of the Methods of Measurement*]), the *Celiang yitong* (測量異同 [*Similarities and Differences in Measurement*]) and the *Gougu yi* (句股義 [*Principles of Right Triangle*]), with a view to applying the Euclidean method to Chinese mathematics. At the beginning of his preface to the *Jihe yuanben*, he says that Chinese mathematics goes back to the legendary period of the sage rulers Yao (尭) and Shun (舜) but that the old tradition of learning perished at the Burning of Books (焚書, 213 B.C.) under the

26 Cf. J.-C. Martzloff, *Histoire des mathématiques chinoises*, Paris, 1987, p. 100; Engelfriet, *op. cit.* (n. 3), p. 140.

27 *Xu Guangqi quanji* (徐光啓全集), vol. 4, p. 9.

28 Engelfriet, *op. cit.* (n. 3), pp. 141–2.

despotic rule of the First Emperor Qin Shi Huang.[29] Yet in reality, the *Jiuzhang suanshu* just cited above is said to have appeared later than 100 B.C. and its commentary was made by Liu Hui (劉徽), a famous mathematician in the third century A.D.[30] The *Zhoubi suanjing* (周髀算経 [*The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven*]), an older text of Chinese astronomy and mathematics, had also its commentaries composed through the period from Later Han (後漢, 25–220 A.D.) to Tang (唐).[31] Accordingly Xu must have had some knowledge of these classical works. As a matter of fact, in his preface to the *Celiang fayi* which is a translation of Clavius' *Geometria Practica*, he referred to them in order to explain why Euclid's work is indispensable to Chinese mathematics:

> *Yi* (義) combines the methods but only began in 1607. Why did we have to wait until so late? Because the six books of the *Jihe yuanben* were first completed in that year and it is only since then that it has become possible to transmit *yi*. In terms of method, is there any difference between this work and the *Jiuzhang suanshu* or the *Zhoubi suanjing* in treating a right triangle or measurement of distance? No, there is no difference. But if there is no difference, what is valuable in this work? What is valuable is just its *yi*. In the old days the schools of Liu Hui and Shen Gua were all able to talk about measurement of distance. They could explain by drawing a diagram but could not do so by putting two diagrams. … Why? Because there were no grounds to do so. … The *Zhoubi suanjing* do not say whether there are any grounds or not. If any, their grounds would need another ones and their explanations should come to no end, if it were not for the *Jihe yuanben*.[32]

In China, since very old times, the special procedures of calculation and solving mathematical problems, generally called algorithm, have been practiced and highly developed. Xu takes here a critical view of the traditional mathematics and states that it can indeed supply the methods (法) for the solution to problems but seriously lacks "yi", which can be provided only by the *Jihe yuanben*. Obviously "yi" indicates here a logical process of establishing the truth of the reasons why a solution can be reached and of making sure the results of calculation in the most objective manner. Xu emphasizes the fact that this scientific process, which signifies the system of proving mathematical propositions deductively, was introduced to China for the first time by the publication of the *Jihe yuanben*. As mentioned above, Matteo Ricci has already noted in his preface that this work can clarify "why mathematics gets what it gets (明幾何之所以然)". Xu Guangqi's specific purpose here is to point out an important defect of Chinese mathematics more exactly and to propose a most certain and effective remedy for it. Considering that he is not a professional scholar of mathematics

29 *Xu Guangqi quanji* (徐光啓全集), vol. 4, p. 4.

30 Cf. Li Yan (李儼), *History of Chinese Mathematics* (支那数学史), Tokyo, 1940, pp. 13–7; K. Yabuuchi (藪内清), *Chinese Mathematics* (中国の数学), Tokyo, 1974, pp. 43–52.

31 Cf. Li, *op. cit.* (n. 30), pp. 9–10; J. Needham, *Science and Civilisation in China* (中国の科学と文明), vol. 4, Tokyo, 1975, pp. 28–33.

32 *Xu Guangqi quanji* (徐光啓全集), vol. 5, p. 5.

like Liu Hui, we shall probably wonder why Xu was so actively involved in mathematical studies that he not only translated the *Elements* and Clavius' work but even wrote the *Celiang yitong* and the *Gougu yi* to make passionate attempts to apply a deductive demonstration to some models of Chinese algorithmic solutions. This unusual inclination to Euclid cannot be comprehended if we do not fully recognize that he was just not a simple mathematician but a public man of Ming China who was deeply concerned in the affairs of his times.

During the late Ming period the Chinese state and society confronted the urgent problems of national defence and agriculture. China was then threatened by the foreign military powers of both Japanese pirates known as Wokous (倭寇) and a northeastern people of Manchuria. In addition, for all successful overseas trading by merchants, the farming villages, throughout the nation, were too exhausted by frequent famines and natural disasters to support the total population of two hundred and fifty million. In this time of domestic crises the traditional learning and education inevitably had to experience new intellectual trends, and the change from old leaning centered on the Confucian classics to "practical or concrete studies" (実学, shixue) was stimulated and accelerated.[33] The Chinese sciences such as mathematics, astronomy and pharmacy, although well advanced from early antiquity, were no longer fit to cope with the actual difficulties of establishing the correct calendar and the surveying methods indispensable to military affairs and agriculture, as well as of developing farmlands and agricultural products. It can be easily understood from this social and political context why Xu was so devoted to the *Elements*. The chief motive that led him to his enthusiastic studies of Western mathematics was that he was convinced of both the academic and social values of the *Jihe yuanben* as something uniquely capable to offer a most firm base to turning Chinese sciences to practical use. In his preface and prelude to this work, he insisted on its necessity and usefulness for not only restoring Chinese mathematics but also working on other sciences and technologies:

> On the whole, what is useless makes what is useful: it makes the basis of a great many things that are useful (蓋不用為用、衆用所基). Truly, this work can be called "the garden of innumerable forms" or "the ocean of hundred schools".
>
> (Preface to the *Jihe yuanben*)[34]

> If one can understand the study of mathematics, his exactness will greatly increase. Hence it should be through this way that the people of this world will be led back to solid practice (実用). (Prelude to the *Jihe yuanben*)[35]

> The usefulness of this book is extremely broad. It is what is most urgently needed in these days. (Prelude to the *Jihe yuanben*)[36]

Actually, with these ideas in his own mind, Xu Guangqi undertook, after the

33 Cf. Okamoto, art. cit. (n. 6), pp. 49–52.
34 *Xu Guangqi quanji* (徐光啓全集), vol. 4, p. 4.
35 *Xu Guangqi quanji* (徐光啓全集), vol. 4, p. 12.
36 *Xu Guangqi quanji* (徐光啓全集), vol. 4, p. 13.

translation of the *Elements*, to resolve the pressing matters of internal administration coming up in the domains of water control, farming, manufacture of new weapons and revision of calendar. Seeking successively the technical improvements and reforms in those areas, he looks to have offered his remaining life entirely to national welfare and military defence, no matter to what extent he could succeed at each attempt. When viewed from this broader perspective of his words and achievements, Xu's overall interpretation and evaluation of Euclid's mathematics seems to emerge as mainly based on the traditional morality rooted in Confucianism. In late Ming and early Qing China a current of thought called 経世致用 (jingshi zhiyong [ordering the world and promoting utility]) was born among Confucian scholars and, holding up such a new ideal, they proposed that learning should be used for improving the actual—chiefly economic and military—conditions of society. Xu's works and career can be counted just among the foremost initiatives in forming this intellectual movement.[37] Even though he became a Christian, he always manifested much more interest in social problems of the present world than in man's fate after death. A similar type of concern can be recognized in his teacher and co-translator. Matteo Ricci, as a Catholic missionary, was more interested in utilizing Euclidean mathematics for the propagation of Christianity than in diffusing the scientific work itself, thinking that, the more objectively he could present the world view of the Western religion by backing it up with mathematical and scientific knowledge, the more Chinese people would become persuaded into believing its spiritual doctrine.[38] In sum, both translators conceived, from the beginning or on the way of their work, the same idea of making the best use of mathematics for each different kind of purpose.

In the preceding chapter, I have left open a linguistic question about the word 論. But now, when we return to the problem after some sketch of the social and cultural context drawn above, it seems that the enigma will be resolved. It is certain that the translation of "mathematical demonstration" as 論, a word with many meanings like "discussion", "opinion" etc., is not only inexact but almost contradicts Euclid's idea of the rigorous proof which does not allow any personal, arbitrary discussion. Why, instead of the ambiguous word 論, a more adequate technical term such as 証 (zheng), 弁 (bian), 験 (yan) was not adopted?[39] A simple but most probable reason would be, in my view, that Ricci and Xu shared a strong sense that the procedures or results of demonstration should not be confined within a mathematical book and narrowly restricted to a specific domain. In his preface to the *Jihe yuanben* Ricci has enumerated the applied sciences, essential for social life and administration, like astronomy, meteorology, study of calendar, surveying, architecture, mechanical engineering and geography, in order to emphasize the utility of mathematics to them:

37 Cf. P. Engelfriet & M.-K. Siu, Xu Guangqi's Attempt to Integrate Western and Chinese Mathematics, in: Jami, Engelfriet & Blue, *op. cit.* (n. 5), p. 308.

38 Cf. Engelfriet, The Chinese Euclid and its European Context (art. cit., n. 17) pp. 111–35.

39 Cf. Martzloff, art. cit. (n. 17), p. 174: n. 9; Engelfriet, *op. cit.* (n. 3), pp. 149–50.

> 此類皆幾何家正属矣。若其余家、大道小道、無不藉幾何之論以成其業者。[40]
> (All these kinds of skills rightly belong to the realm of mathematicians. Moreover, any other specialists, no matter what differences of importance may exist as to their professions, cannot do their work without depending on theory of mathematics as its basis.)

Here, "theory of mathematics" (幾何之論) may be interpreted in a general sense, but, as far as the *Jihe yuanben* is concerned, it means exactly 論 of the propositions, that is to prove them incontestably true. In the preface to the *Jihe yuanben* by Xu, Ricci is also said to have referred, just as 論, to "depending on theory of mathematics as the basis" of other disciplines:

> 独謂此書未訳、即他書俱不可得論。[41]
> ([The Master] only said: As far as this book has not yet been translated, it will be impossible to give a demonstration to any other book.)

Xu Guangqi had asked him at first to translate a Western work on astronomy, but Ricci's opinion was that the *Elements* was to be translated first of all. In the reason quoted above, what gives a logical consistency to other scientific works and proves their various theories is called 論. Besides, Xu himself writes in the last part of the same preface: "Although [this work] actually has not yet completed, with it as a reference, it is already possible to provide other books with proofs " (雖実未竟、然以当他書、既可得而論矣).[42] As the word 論 has multiple senses, it is also possible to read each *lun* in the three passages cited above as meaning "discuss", "comment", "criticize" and so on. From these considerations we seem to have reached a conclusion: it is just because 論 is multivocal, a word quite commonly used and open to many contexts, that in their translation Ricci and Xu decided to choose it as the term for mathematical demonstration. Apparently, this deliberate choice reveals their conscious intention to impress the contemporary Chinese with the universal applicability of Euclidean mathematics.

But afterwards, it proved to be another matter whether their scheme could work well as originally designed. On one side, the growing trend of "practical studies" (実学) greatly encouraged the *Jihe yuanben* to attract general attention, so that the knowledge of geometrical studies spread fairly soon through China. On the other, however, the fundamental nature of Euclid's logic of deduction and his axiomatic system faced a considerable difficulty of being understood accurately. To make matters worse, the mathematical system of Euclid even suffered serious distortion and was received in a manner almost contrary to the translators' intention.

Firstly, although Xu Guangqi himself attempted, in his *Celiang yitong* and *Gougu yi* mentioned above, to transfer the consecutive arithmetical operations of algorithm (addition, subtraction, multiplication and division) into geometrical constructions and

40 *Xu Guangqi quanji* (徐光啓全集), vol. 4, p. 8.
41 *Xu Guangqi quanji* (徐光啓全集), vol. 4, p. 4.
42 *Xu Guangqi quanji* (徐光啓全集), vol. 4, p. 4.

thus to demonstrate "geometrically" the exactness of traditional Chinese mathematics, he failed, on the whole, to built up, by the deductive method, the explanations clear enough to show what the mechanism of algorithm was founded on.[43] Occasionally his demonstrations with geometrical figures became too complicated and, in some cases, he needed for solution to bring forth even the old method of "rule of three" (三数法) contained in the *Jiuzhang suanshu*.[44]

Secondly, soon after the publication of the *Jihe yuanben*, there appeared, one after another, the handbooks of geometry (for example, 幾何体論 [*Jihe tilun*] by 孫元化 [Sun Yuanhua], 1608; 数度衍 [*Shu duyan*] by 方中通 [Fang Zhongtong], 1661; 幾何易簡集 [*Jihe yijianji*] by 李子金 [Li Zijin], 1679; 幾何通解 [*Jihe tongjie*] by 梅文鼎 [Mei Wending], 1692; 幾何論約 [*Jihe lunyue*] by 杜知耕 [Du Zhigeng], 1700; 幾何原本挙要 [*Jihe yuanben juyao*] by 荘亨陽 [*Zhuang Hengyang*], before 1746),[45] yet in those books, the demonstration of propositions, even though it constitutes the very core of the *Elements*, was totally eliminated or considerably abbreviated.[46] In the text of Euclid, the demonstration tends to become longer and more intricate as it proceeds with the combination of more axioms and more proved propositions or theorems. This course of rigid and thorough proof was very likely to look unnecessarily circuitous to the Chinese readers accustomed to simpler explanation with sight and number. Besides, it should not be overlooked that in their source book, the *Jihe yuanben*, the mathematical demonstration was presented as 論: this seems to have had the opposite effect of what had been intended. For instance, in the preface to the *Jihe lunyue* Wu Xuehao (呉学顥) said: 幾何一書絶非其論 (This book of geometry [i.e. the *Jihe yuanben*] has nothing to do at all with that kind of theories).[47] This brief comment suggests not only that in actual fact the parts of demonstration in the *Elements*, because of their difficulty, came to be closely related with the abstruse and harmful doctrines of foreign religions,[48] but also that such a popular prejudice was partly based on the ambiguous term 論. It is a very ironical turn of events that the word 論, chosen with the view of making Western mathematics fit for practical use, helped to strengthen a vulgar association with the mysterious Christian thought that looked similar to Buddhism and, consequently, hindered the true scientific contribution of the *Elements* to Chinese mathematics.

The interpretation of "demonstration" as 論 thus worked to bring about an unfavorable situation and to alienate Chinese people from Euclid's deductive method. However, its effects were not limited to the adverse popular reaction. It is the last and worst consequence that some professional scientists, stimulated by the word 論,

43 Cf. Engelfriet & Siu, art. cit. (n. 37), pp. 291–310.

44 *Xu Guangqi quanji* (徐光啓全集), vol. 5, p. 37.

45 Cf. Li, *op. cit.* (n. 30), pp. 170–1, 186–7; J.-C. Martzloff, La compréhension chinoise des méthodes démonstratives euclidiennes au cours du XVII^e siècle et au début du XVIII^e siècle, in: *Actes du II^e Colloque Internationale de Sinologie. Les rapports entre la Chine et l'Europe au temps des lumières*, pp. 125–43, Paris, 1980; Qian Baocong (銭宝琮), *History of Chinese Mathematics* (中国数学史), Tokyo, 1990, p. 245.

46 Cf. Martzloff, art. cit. (n. 45).

47 Cited in: Engelfriet, *op. cit.* (n. 3), pp. 463–4.

48 Cf. Martzloff, art. cit. (n. 17), pp. 172, 176: n. 38; J.-C. Martzloff, Space and Time in Chinese Texts of Astronomy and Mathematical Astronomy in the Seventeenth and Eighteenth Centuries, *Chinese Science* 11 (1993/94), p. 71.

started to make a reappraisal of the *Elements*. Mei Wending (梅文鼎, 1633–1721), a celebrated mathematician and astronomer in the early Qing period, was the most prominent figure among those critical specialists. After he closely studied the *Jihe yuanben*, Mei wrote the *Jihe tongjie* (幾何通解). In this work, he tried to show that the sixteen propositions of Euclidean geometry could be fully elucidated by applying the theory of gougu (句股), which was the Chinese equivalent to the Pythagorean theorem concerning right triangle and had been explained in detail in the *Zhoubi suanjing* and the *Jiuzhang suanshu*. His conclusion was: "There is no mention of gougu in the *Jihe yuanben*, but all its theories can be proved by gougu (幾何不言句股。然其理並句股也)".[49] As I remarked above, Xu Guangqi sharply pointed out that traditional Chinese mathematics was totally devoid of strict proof and proposed the urgent necessity of making up the fatal deficit with Euclid's work. Against this view, Mei Wending asserted that Chinese mathematics had already established, since very old times, the methods of solving the Euclidean propositions in a plainer and simpler way. It is noteworthy that Mei, deeply aware of the usage of 論 in the *Jihe yuanben* and Xu's critical comment on traditional mathematics, entitled as 論 his own treatise based on Chinese method (cf. 方程論序 [Preface to the *Fangcheng lun*]).[50] But what constituted 論 in his own work was not anything similar to the rigorous proofs of the *Elements*. On the contrary, the key term 論, employed by Mei in a more general sense of "theory" or "discourse", gave him a legitimate pretext for criticizing Euclid's mathematics.

Mei Wending's solutions to geometrical problems were visually constructed enough to be readily understood like those offered in the commentary by Liu Hui on the *Jiuzhang suanshu*. But there was no such strict demonstration as in Euclid.[51] In spite of this essential difference, Mei, after making a comparative study of Western and Eastern astronomies, firmly advocated a theory of "Chinese origins of Western learning" (西学中源 [xixue zhongyuan]) and expounded that Western astronomy had its rise in China.[52] Thus he proposed the critical reception of Western science in a perspective centered on ancient Chinese scholarship. Just as Ricci and Xu he laid great stress on the applicability of mathematics but, unlike them, less on how logically true it should have been as a pure and exact science. A reversal Mei's further "pragmatic"[53] evaluation of the *Elements* caused in the positions and roles of Western and Chinese mathematics had a direct influence on contemporary intellectuals in China. Actually, in the next period of the Qing dynasty, what was mainly promoted for mathematics was not the advanced studies of a creative nature but the historical works of restoring ancient Chinese sciences, such as discovering and editing mathematical classics for the *Siku quanshu* (四庫全書 [*The Complete Collection of the Four Treasuries*])

49 Cf. K. Hashimoto (橋本敬造), Mathematical Studies of Mei Wending (梅文鼎の数学研究), *Toho Gakuho: Journal of Oriental Studies* 44 (1973), pp. 265–72; Yabuuchi, *op. cit.* (n. 30), p. 162; Qian, *op. cit.* (n. 45), p. 273; Engelfriet, *op. cit.* (n. 3), pp. 407–21.

50 Partly quoted in: W.-Sh. Horng, The Influence of Euclid's *Elements* on Xu Guangqi and his Successors, in: Jami, Engelfriet & Blue, *op. cit.* (n. 5), p. 386.

51 Cf. Martzloff, art. cit. (n. 45), pp. 134–42; J.-C. Martzloff, La géométrie euclidienne selon Mei Wending, *Historia Scientiarum* 21 (1981), pp. 27–42.

52 Cf. Yabuuchi, *op. cit.* (n. 30), p. 163–4; An, *op. cit.* (n. 12), p. 251–6.

53 Hashimoto, *op. cit.* (n. 49), p. 273.

and compiling the biographies of astronomers and mathematicians in the *Chourenzhuan* (疇人伝, by Ruan Yuan 阮元).[54]

4. Conclusion: China and Japan

In these conditions Euclid's deductive system and its way of logical thinking, whose nature was quite different in itself for the Chinese, faded away more and more into the mist. From these circumstances it can be easily understood that, conversely to the numerous new terms of geometrical figures still in common use, the basic terminology concerning a process of proof created in the *Jihe yuanben* (界説, 求作, 公論, 題, 論) was utterly forgotten in later ages.[55] And when we think about why such a result was yielded, the part played by the polysemous word 論, which the translators used for Euclid's demonstration to give a positive impression of the practical utility of mathematics, should be seen, as I have shown above, as least negligible.

Certainly, for Xu Guangqi, it was a most pressing matter to apply Western mathematics to "practical studies" (実学), considered from his social position as well as from his Confucian ethical standpoint. But quite unexpectedly, it was just his great eagerness to contribute to "solid practice" (実用) that worked later as an underlying cause for urging the return to old tradition and, consequently, for delaying the modernization of Chinese science. In fact, when the *Jihe yuanben* was completed at last in 1859, China, as a result of the Opium War (1840–2), was becoming a half-colony of the Western Great Powers. Afterwards China, in terms of modernization of science, had a much later start than Japan quickly westernized after the Meiji Restoration (1868) and was defeated in the Sino-Japanese War (1894–5).

On the other hand, in Japan, "wasan" (和算 [Japanese mathematics]), which inherited the Chinese mathematical tradition and developed remarkably during the Edo period, was totally abolished from schools by the government (1872) soon after the Restoration, and much more "useful" Western mathematics replaced it, without delay, under the new educational policy of "practical studies" (実学). Moreover, since the early Dutch studies (蘭学, rangaku) of the Edo period, Japanese translators of Western scientific books had learned a foreign language and generally done their works by reading a Western text themselves. This direct method of translation in Japan differed from the way in which Xu Guangqi translated the *Elements* at Matteo Ricci's dictation in Chinese and which remained for a long time as a typical style of Western studies in China.[56] The prompt and sweeping change from taking one course to another and the firsthand and swift assimilation of foreign cultures, both of which

54 Cf. Li, *op. cit.* (n. 30), p. 192–7; Yabuuchi, *op. cit.* (n. 30), p. 177–97; Li & Du, *op. cit.* (n. 23), pp. 223–33.

55 I have not found any study which recognized this curious phenomenon clearly enough and discussed it in terms of cultural context.

56 Until the middle of the nineteenth century the Chinese used to make a translation of Western book through the dictation of a foreign teacher such as Christian missionary: cf. S. Nagata (永田小絵), Transition and Dynamics of Translation in China during the Qing Dynasty, *Interpretation Studies* 6 (2006), pp. 207–28.

were conspicuous characteristics of the Japanese modernization, make a marked contrast with the Chinese gradual and complex reaction and can be viewed, on the whole, as an effect of having taken full advantage of Japan's geographical and cultural position on the periphery of a great civilization zone and therefore, in a word, of its "rootlessness".

But we could never say that the Japanese today have nothing to learn from the *Jihe yuanben* any more. Its problems with translation and reception do show us what state of affairs can be brought about by trying, without a careful thought, to connect the fundamental studies of science and culture to practical use. Of the true value of Euclid's *Elements* Xu Guangqi said: "On the whole, what is useless makes what is useful: it makes the basis of a great many things that are useful (蓋不用為用、衆用所基)".[57] In Qing China those words were interpreted in such a different and simplistic way that Chinese intellectuals drove themselves to the learning of ancient scientific books which looked to them richer and more useful than Western science. If we turn again our attention from the Asian Continent to the Japanese Islands, where "solid practice" (実用) and "practical studies" (実学) still remain the central matters, there seems to be no better lesson than those past experiences of China. Though ironically, they do suggest to us that the surest way leading to the future is to take a broader, universal view and to make firm and solid the foundations of scientific and cultural researches which are getting fragile and even hollow here and there.[58] Don't we still need, then, to make good *use* of our "fated merit" (or "seeming demerit") of having no original traditions in the past—any more than the vanished "wasan" of continental origin—to be returning to from now on?

57 Cf. the preceding chapter and n. 34.

58 In this connection I might mention that, as far as the Euclid's *Elements* is concerned, after a very simple, abridged version issued for secondary education in the Meiji period (1884), there was a long interval of more than eighty years before the first publication of its complete Japanese translation in 1971 (ユークリッド原論, translated by K. Nakamura and others, Tokyo). This date is also far later—by more than a century—than that of the earliest complete Chinese version (1859), though we shall have now *all the extant works* of Euclid in Japanese (エウクレイデス全集, in five volumes, translated by K. Saito, N. Miura and others, Tokyo, 2008–).

On the interrelations between the traditional and Western mathematics in China and Japan after the middle of the sixteenth century: cf. Xu Zelin (徐澤林), A Comparison between the Chinese and Japanese Mathematics in Modern Times from the Viewpoint of World Mathematical Culture, *RIMS Kôkyûroku* 1444 (2005), pp. 19–28.